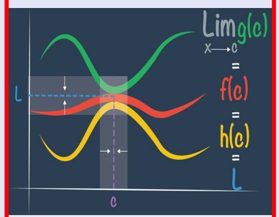
# Calculus I

# Lecture 38




Feb 19-8:47 AM

A rectangular shape has area of $1000\ cm^2$.

$x$ | $1000\ cm^2$ | $y$

$\Rightarrow xy = 1000 \Rightarrow y = \dfrac{1000}{x}$

$10\sqrt{10}$ by $10\sqrt{10}$

Find $\boxed{\text{the dimensions } x \,\&\, y}$ with $\boxed{\text{Smallest perimeter}}$

Minimize $2x + 2y$

Let $f(x) = 2x + 2\left(\dfrac{1000}{x}\right)$ be the function for the perimeter.

$f(x) = 2x + 2000x^{-1}$

$f'(x) = 2 - 2000x^{-2}$

$f''(x) = 4000x^{-3}$

$f''(x) = \dfrac{4000}{x^3}$

Since $x > 0$

$f''(x) > 0$

$f(x)$ is Concave up

Min. Point at $x = 10\sqrt{10}$

$f'(x) = 0$

$2 - \dfrac{2000}{x^2} = 0$

$2x^2 - 2000 = 0$

$x^2 - 1000 = 0$

$x^2 = 1000$

$x = \pm\sqrt{1000}$

$x > 0 \rightarrow x = \sqrt{1000}$

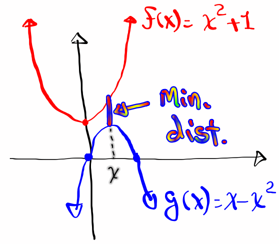
$\boxed{x = 10\sqrt{10}}$

$y = \dfrac{1000}{x} \qquad y = \dfrac{\overset{10}{\cancel{1000}}}{\cancel{10}\sqrt{10}} \cdot \dfrac{\sqrt{10}}{\sqrt{10}} \qquad \boxed{y = 10\sqrt{10}}$

Apr 22-8:46 AM

$\boxed{f(x) = x^2 + 1}$ , $\boxed{g(x) = x - x^2}$ $= x(1 - x)$

Find $x$-value where the vertical distance between the graphs of these two functions is minimum.




$f(x) = g(x)$

$x^2 + 1 = x - x^2$

$2x^2 - x + 1 = 0$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(1)}}{2(2)} = \frac{1 \pm \sqrt{-7}}{4}$

No real Solutions

$\Rightarrow$ No intersections

$V(x) = f(x) - g(x)$

$= x^2 + 1 - (x - x^2)$

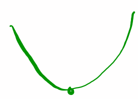
$= x^2 + 1 - x + x^2$

$= 2x^2 - x + 1$

$V'(x) = 4x - 1$

$V''(x) = 4 > 0$

$V(x)$ is C.U.



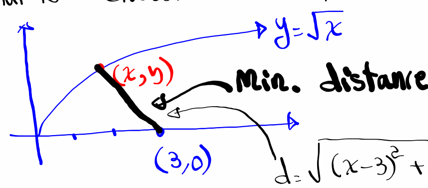
$V'(x) = 0$

$4x - 1 = 0$

$\boxed{x = \frac{1}{4}}$

Apr 22-8:56 AM

Find $\boxed{\text{the point}}$ on the graph of $y = \sqrt{x}$ that is closest to the point $(3, 0)$.




$d = \sqrt{(x-3)^2 + (y-0)^2}$

$d = \sqrt{(x-3)^2 + y^2}$

$d = \sqrt{(x-3)^2 + (\sqrt{x})^2}$

$d = \sqrt{(x-3)^2 + x}$

Let $f(x) = (x-3)^2 + x$

If we minimize $f(x)$, then distance will be minimized.

$f'(x) = 2(x-3)^1 \cdot 1 + 1$

$f'(x) = 2x - 6 + 1 \rightarrow f'(x) = 2x - 5$

$f''(x) = 2 > 0$

$f(x)$ is C.U.

$f'(x) = 0$

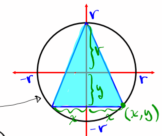
$2x - 5 = 0$

$x = \frac{5}{2}$

Point $\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$

Apr 22-9:06 AM

find the dimensions of the isosceles triangle of largest area that can be inscribed in a Circle of radius r.




$x^2+y^2=r^2$

base $2x$
height $y+r$ $\Rightarrow$ Area $= \frac{bh}{2} = \frac{\cancel{2}x\cdot(y+r)}{\cancel{2}} = x(y+r)$

$y^2 = r^2-x^2$
$y = \sqrt{r^2-x^2}$

Let $f(x)$ be the area

$f(x) = x\left(\sqrt{r^2-x^2}+r\right)$

$= x\left[(r^2-x^2)^{1/2}+r\right]$

$f'(x) = 1\cdot\left[(r^2-x^2)^{1/2}+r\right] + x\cdot\left[\frac{1}{\cancel{2}}(r^2-x^2)^{-1/2}\cdot(-\cancel{2}x)+0\right]$

$f'(x) = \sqrt{r^2-x^2}+r + \frac{-x^2}{\sqrt{r^2-x^2}} = \frac{r^2-x^2+r\sqrt{r^2-x^2}-x^2}{\sqrt{r^2-x^2}}$

$f'(x) = \frac{r^2-2x^2+r\sqrt{r^2-x^2}}{\sqrt{r^2-x^2}}$

$f'(x)=0$ when num.$=0$
$f'(x)$ is und. when Deno.$=0$

$r^2-2x^2+r\sqrt{r^2-x^2}=0$

$r\sqrt{r^2-x^2} = 2x^2-r^2$

$\left(r\sqrt{r^2-x^2}\right)^2 = \left(2x^2-r^2\right)^2$

$r^2(r^2-x^2) = 4x^4-4r^2x^2+r^4$

$\cancel{r^4}-r^2x^2 = 4x^4-4r^2x^2+\cancel{r^4}$

$4x^4-3r^2x^2=0$

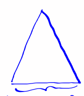
$x^2(4x^2-3r^2)=0$

$x=0$

$4x^2-3r^2=0$

$x^2 = \frac{3r^2}{4}$

$x = \frac{r}{2}\sqrt{3}$



$b = 2x = 2\cdot\frac{r}{2}\sqrt{3} \Rightarrow \boxed{b = r\sqrt{3}}$

height $= r+y = r+\sqrt{r^2-x^2} = r+\sqrt{r^2-\frac{3r^2}{4}}$

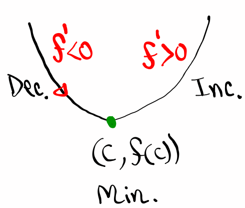
$= r+\sqrt{\frac{1}{4}r^2} = r+\frac{1}{2}r = \frac{3}{2}r$ $\quad \boxed{h = \frac{3r}{2}}$

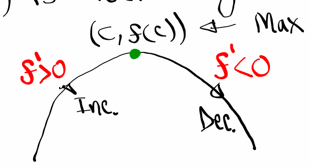
Apr 22-9:16 AM

First Derivative Test

$f'(x) < 0 \rightarrow f(x)$ is decreasing

$f'(x) > 0 \rightarrow f(x)$ is decreasing




Second Derivative Test

If c is a C.N. $\rightarrow f'(c)=0$ or undefined

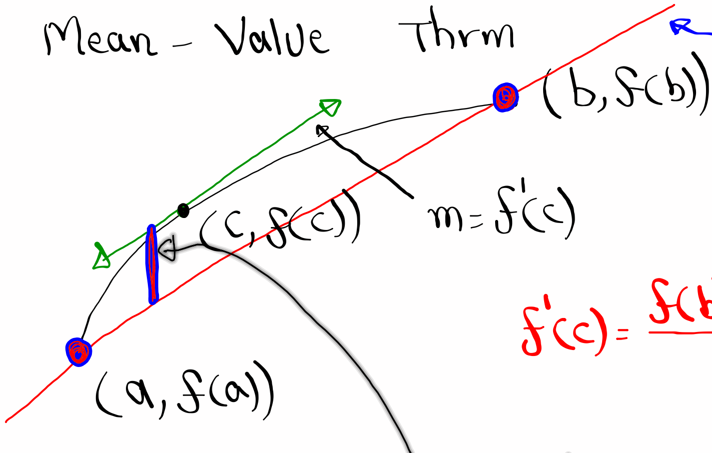
$f''(c) > 0 \rightarrow$ C.U. $\rightarrow$ Min. value

$f''(c) < 0 \rightarrow$ C.D $\rightarrow$ Max. Value

Google First & Second Derivative Tests.

Apr 22-9:44 AM

Mean - Value Thrm




$m = \dfrac{f(b) - f(a)}{b - a}$

$f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Vertical distance

$y_{\text{Tan. line}} - y_{\text{secant line}}$

Google MVT

Apr 22-9:51 AM